

UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF PHYSICS & ENGINEERING PHYSICS

PHYSICS 251.3

Final Examination

Saturday, Dec. 18, 2004

**WRITE YOUR NAME AND STUDENT NUMBER ON THE
EXAMINATION BOOKLETS!**

You **need** an electronic calculator.

You **may** use four letter size *pages* of self-prepared notes.

Some constants (which you may or may not need):

Speed of light in vacuum:

$$c = 2.998 \times 10^8 \text{ m/s}$$

Absolute value of electron charge:

$$e = 1.602 \times 10^{-19} \text{ C}$$

Electron mass:

$$m_e = 9.109 \times 10^{-31} \text{ kg} = 511.0 \text{ keV}/c^2$$

Compton wavelength of the electron:

$$\lambda_C = \frac{h}{m_e c} = 2.426 \text{ pm} = 2.426 \times 10^{-12} \text{ m}$$

Bohr magneton:

$$\mu_B = 5.788 \times 10^{-5} \text{ eV/T}$$

Atomic mass unit:

$$1 \text{ u} = 931.5 \text{ MeV}/c^2$$

Planck's constant:

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$hc = 1.986 \times 10^{-16} \text{ J} \cdot \text{nm} = 1240 \text{ eV} \cdot \text{nm}$$

$$\hbar = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 6.582 \times 10^{-16} \text{ eV} \cdot \text{s}$$

$$\hbar c = 3.161 \times 10^{-17} \text{ J} \cdot \text{nm} = 197.3 \text{ eV} \cdot \text{nm}$$

Boltzmann's constant:

$$k_B = 1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K}$$

Stefan-Boltzmann constant:

$$\sigma = 5.6705 \times 10^{-8} \text{ W}/(\text{m}^2 \text{K}^4)$$

The constant in Wien's displacement law is $2898 \mu\text{m} \cdot \text{K}$.

$$\hbar c = 197.3 \text{ eV} \cdot \text{nm}$$

1. X-rays of energy $E_0 = 112.5 \text{ keV}$ are scattered from a block of carbon.

1a. Under which scattering angle ϑ would we find scattered photons with wavelength $\lambda = 12.14 \text{ pm}$? (3 marks)

1b. How much kinetic energy did a photon from 1a transfer to an electron? (2 marks)

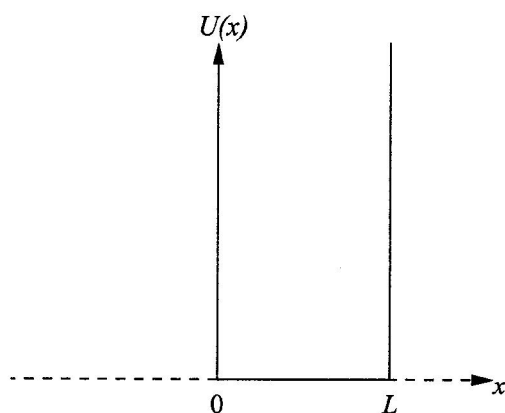
1c. How large is the de Broglie wavelength of the scattered electron? (3 marks)
(You can calculate non-relativistically.)

$$K = p^2 / 2m$$

$$\sqrt{2mK} = p$$

2. Aluminum has a work function of 4.08 eV . An aluminum electrode is irradiated with monochromatic radiation. The maximal kinetic energy of photoelectrons emitted from the electrode is measured as $K = 5.21 \text{ eV}$. How large is the momentum of the incident photons? Give the result in units of eV/c . (3 marks)

3. An electron is confined to a box of width L . We consider this electron only in one dimension, and we assume that the potential energy of the electron vanishes inside the box and diverges outside of the box ("infinite square well"):



$$U(x) = \begin{cases} 0 & \text{if } 0 < x < L, \\ \infty & \text{if } x < 0 \text{ or } x > L. \end{cases}$$

3a. Write down the time-independent Schrödinger equation for the electron inside the square well. (2 marks)

3b. Find the general solution for the equation from 3a. (4 marks)

3c. Which boundary conditions does the wavefunction of the electron have to fulfill at $x = 0$ and $x = L$?

The wavefunction has to be continuous at infinite jumps, but not smooth (smoothness is only required at finite jumps).

What do the boundary conditions tell you about the allowed energy values of the electron in the box? (4 marks)

3d. The square well has width $L = 2.52 \text{ \AA}$. How large is the ground state energy of the electron in units of eV? (2 marks)

3e. Estimate the momentum uncertainty Δp of the electron in the square well. Give the result in units eV/c.

Remark: This estimate requires a *very short* calculation. You do **not** need to calculate any momentum expectation values or integrals! (2 marks)

4. The ground state energy of hydrogen is $E_1 = -13.6 \text{ eV}$.

4a. What are the allowed angular momentum quantum numbers (also called orbital quantum numbers) ℓ for principal quantum numbers $n = 2$ and $n = 4$? (3 marks)

4b. Which transitions from initial quantum numbers $n = 4$ and ℓ to final quantum numbers $n' = 2$ and ℓ' are allowed by the selection rules? (Do not worry about magnetic quantum numbers in this part. I am only asking for the allowed transitions $\ell \rightarrow \ell' = \ell + \Delta\ell$.) (4 marks)

4c. How large are the energy E and angular momentum L of an electron in a 4d state? Give the results in units of eV and eV·s, respectively. (4 marks)

4d. A photon is emitted in a transition between a 4d and a 2p state. How large is the frequency of the emitted photon? (4 marks)

4e. Now we switch on an external magnetic field $B = 3.65 \text{ T}$. Draw a schematic energy diagram showing the 2p and 4d levels *before* and *after* the magnetic field is switched on. (Neglect any effects from spin on the energy levels.) (3 marks)

4f. In the energy diagram with the external magnetic field switched on, indicate all the allowed optical transitions. (6 marks)

4g. How many lines in an emission spectrum would you get from 4d to 2p with the external magnetic field turned on?

How large is the frequency split Δf between these lines?

(Note: I am explicitly asking for the split in *frequency*, not *angular frequency*.)

(3 marks)

$\Sigma = 52 \text{ marks}$